MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF AERONAUTICAL ENGINEERING
CAMBRIDGE 39, MASSACHUSETTS

INSTRUMENTATION LABORATORY
C. S. DRAPER
DIRECTOR
A. F. COLEMAN
EXECUTIVE OFFICER

HOOD BUILDING
D. I. C.

1 June 1948

Capt. Everett E. Mann
Room 5-233

Dear Sir:

Com. E. S. Keats prepared his master thesis, The Turning Performance of Airplanes, under my supervision. The subject matter of this thesis was familiar to him by virtue of his experience as a Naval aviator. For this reason he was able to delve more deeply into the problem than the average student and hence his contribution was more profound. I know that the material in his thesis will be a valuable reference for the government projects under way in our laboratory, and feel sure that his thesis will have a bearing on the tactical use of existing and future aircraft. It was a pleasure to have a man of his experience and enthusiasm as a student.

My only criticism of his thesis involves the presentation of material. The primary results of his investigation should have been discussed in the body of the report, relegating the mathematical analyses to the appendices. He is planning to try his new idea in flight in order to check his theoretical conclusions. When he does, his work should make an extremely worthwhile report.

His thesis was of honor level, and hence his Institute grade was H.

THE TURNING PERFORMANCE OF AIRPLANES

BY

EDGAR S. KE[illegible]

B.S., University [illegible] N[illegible]m

193[illegible]

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

AT

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1[illegible]

May 19, 1948

Professor Joseph S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Professor Newell:

In accordance with the regulations of the faculty, I hereby submit a thesis entitled, "The Turning Performance of Airplanes", in partial fulfillment of the requirements for the degree of Master of Science in Aeronautical Engineering.

Respectfully submitted,

TABLE OF CONTENTS

FIGURES

OBJECT

1. To investigate the turning performance of airplanes employing the concepts of radius of turn and angular velocity of turn as criteria.

2. To analyse the results in terms of airplane design parameters and airplane flight parameters.

3. To consider successively steady turns in level flight, turns in a spiral, and turns in non-steady flight.

4. To make calculations applicable to both propeller driven airplanes and turbo-jet airplanes.

INTRODUCTION

A. IMPORTANCE

Fighter airplanes shoot their guns almost invariably in turning maneuvers. The turning performance of a fighter is a measure of its ability to bring its guns to bear on its target.

When one fighter opposes another fighter the engagement often results in a tail chase in circular flight resembling a dog fight. The turns of both fighters are steady turns in level flight or a spiral. These are turns of nearly minimum radius where the fighter with the maximum angular velocity can gain the favorable firing position astern of his opponent.

When a fighter opposes a bomber the fighter executes attacks in which he approaches the bomber in a flight path during which his guns are always pointed slightly ahead of the bomber. This maneuver requires the fighter to fly in turning flight except for the rare cases of approach from directly ahead or astern of the bomber. The turning flight of the fighter is entered into from straight flight and is a non-steady turn.

Although other tatics not requiring turns are possible with existing weapons, and may be required by new weapons, these turning maneuvers have been employed during two World Wars and are in general favor at present.

The analysis of turning performance is, therefore, necessary in determining the design of fighter airplanes, and once they are built, in flying them to the best advantage.

B. HISTORICAL

Although the turning performance of airplanes is touched upon briefly in most general text books in aeronautics, the published literature in the United States devoted exclusively to the subject is limited. So far as can be determined, there is no published literature on non-steady turns.

The first, and undoubtedly the best, monograph on the subject was written in German by E. Salkowski and reprinted by the National Advisory Committee for Aeronautics. The concept of the non-dimensional performance parameter is developed in this report, and by that means a systematic theory of turning flight is built up. Salkowski did not answer many of the pressing questions of turning performance but he showed the way to finding the answers.

One British paper on this subject was printed in 1932, but except for this the period between wars was devoid of turning performance literature. During the last war interest in fighter turning performance at high altitudes was evidenced. The prospect of improving turning performance by the use of flaps also received some attention.

Unfortunately for this subject, other matters influenced fighter design more than turning performance. The published investigations were incomplete and inconclusive. There is no evidence of flaps being designed specifically for improvement of turning. Flaps were designed for landing and take off and only occasionally used for turning.

SYMBOLS

LETTER	DEFINITION	UNITS
T	Thrust of jet or propeller	pounds
D	Total drag of airplane	pounds
L	Total lift of airplane	pounds
C_L	Lift coefficient of airplane	----
C_D	Drag coefficient of airplane	----
V	True Velocity of airplane relative to the air	ft/second
S	Wing area of airplane	ft^2
W	Weight of Airplane	pounds
d	Density of air at the altitude under consideration	$slugs/ft^3$
R	Radius of curvature of the flight path	ft
F_c	Centrifugal force in the turn	pounds
F_r	Vector sum of Centrifugal force and weight of airplane "Resultant Force"	pounds
a	Accelleration of the airplane along its flight path	ft/sec^2

LETTER	DEFINITION	UNITS
A_B	Angle of bank	radians
A_D	Inclination of the flight path to the horizontal "angle of dive"	radians
$\bar{w}$	Angular velocity	radians/sec
g	Newton's constant of proportionality	ft/sec^2
B	Ratio of thrust to weight of airplane. "Inverse Thrust Loading."	----
N	Ratio of resultant force to weight of the airplane. "Accelleration in gravities units".	----
P	Power output of an engine-propeller combination at a given altitude.	ft-lbs/sec
K	Parameters of propeller driven airplanes, level flight.	----
K_s	Parameters of propeller driven airplanes, spiral.	----
V_z	Component of true velocity in the vertical direction, positive downward.	ft/sec
T_j	Thrust of a turbo-jet engine at a given altitude.	lbs

ASSUMPTIONS

(1) That no slip or skid exists.

(2) That thrust and drag act in a direction tangent to the flight path.

(3) That the power output of the engine is the same in a turn as in straight flight, all other things being constant.

(4) That the polar of the airplane is independent of altitude and velocity.

(5) That there is no effect on the polar due to engine power.

(6) That the airplane polar represents a continuous functional relationship between C_L and C_D.

(7) That the elevators are large enough to make the airplane fly at any angle of attack up to the stall.

(8) That the thrust in a propeller driven airplane is inversely proportional to the velocity.

(9) That the thrust in a jet airplane is independent of the velocity.

CHAPTER I

GENERAL EQUATIONS FOR A STEADY TURN.

The airplane in a steady turn in the horizontal plane will be considered first:

The equations of motion are:

$$T = D = \frac{d}{2} V^2 S C_D \text{ lbs} \tag{1}$$

$$L = \left[W^2 + F_c^2\right]^{\frac{1}{2}} = \frac{d}{2} V^2 S C_L \text{ lbs} \tag{2}$$

$$F_c = \frac{W}{g} \frac{V^2}{R} \text{ lbs} \tag{3}$$

Solving for R

$$\frac{W}{g} \frac{V^2}{R} = \left[\left[\frac{d}{2} V^2 S C_L\right]^2 - W^2\right]^{1/2}$$

$$R = \frac{W}{g} \frac{V^2}{\frac{d}{2} V^2 S \left[C_L^2 - \frac{W^2}{(\frac{d}{2} V^2 S)^2}\right]^{1/2}}$$

$$R = \frac{W/S}{\left(\frac{d}{2}\ g\right)\left[C_L^{2} - \frac{W^2}{\left(T/C_D\right)^{2}}\right]^{1/2}}$$

$$R = \frac{W/S}{\left(\frac{d}{2}\ g\right)\left\{C_L^{2} - C_D^{2}\left[\left(\frac{W}{T}\right)\right]^{2}\right\}^{1/2}} \text{ ft.} \tag{4}$$

Solving for $\bar{w}$

$$\bar{w} = \frac{V}{R} = \frac{V\left(\frac{d}{2}\ g\right)\left\{C_L^{2} - C_D^{2}\left[\left(\frac{W}{T}\right)\right]^{2}\right\}^{1/2}}{W/S}$$

$$\bar{w} = \left[\frac{\frac{d}{2}\ g^{2}}{W/S}\right]^{1/2}\left[\left(\frac{T}{W}\right)\frac{C_L^{2}}{C_D} - C_D\left(\frac{W}{T}\right)\right]^{1/2} \text{rad/sec} \tag{5}$$

These equations hold true for any type of power plant.

CHAPTER II

PROPELLER DRIVEN AIRPLANES IN LEVEL FLIGHT

For a propeller driven airplane

$$T = \frac{P}{V} \quad \text{lbs} \tag{6}$$

where P is the power output of the engine propeller combination at any altitude in ft - lbs/sec

Combining with equation (1) yields

$$V = \left[\frac{P}{\frac{d}{2} S \, C_D} \right]^{1/3} \quad \text{ft/sec} \tag{7}$$

$$T = \left[P^2 \frac{d}{2} S \, C_D \right]^{1/3} \quad \text{lbs} \tag{8}$$

$$\frac{T}{W} = \left[\frac{P^2 \frac{d}{2} S}{W^3} \right]^{1/3} C_d^{\;1/3} \tag{9}$$

Let

$$\left[\frac{P^2 \frac{d}{2} S}{W^3} \right]^{1/3} = K \quad \text{a non-dimensional parameter.} \tag{10}$$

Rewriting equations (4) and (5) in terms of equations (9) and (10)

$$R = \frac{W/S}{\left(\frac{d}{2}\, g\right) \left[C_L^{\,2} - \frac{C_D^{\,4/3}}{K^2} \right]^{1/2}} \quad \text{ft} \tag{11}$$

$$\bar{w} = \left[\frac{\frac{d}{2}\, g^2}{W/S} \right]^{1/2} \left[K \frac{C_L^{\,2}}{C_D^{\,2/3}} - \frac{C_D^{\,2/3}}{K} \right]^{1/2} \quad \text{rad/sec} \tag{12}$$

From equation (11) it is seen that the radius of turn is governed by the wing loading, the air density, and the quantity $\left[C_L^{\,2} - \frac{C_D^{\,1/3}}{K^2} \right]$ which may be called the characteristic quantity for the sharpest turn. This characteristic is a function of the airplane polar and the parameter K which is itself a function of density, wing loading, and power loading.

If curves of the characteristic quantity versus C_D are plotted for various values of K they appear as Fig. (2)

Since the minimum R corresponds to the maximum $\left[C_L^2 - \frac{C_D^{4/3}}{K^2}\right]$ it is advantageous to fly at the values of C_D as shown by the heavy line on the graph.

The value of K which is just sufficient to give a zero value of the characteristic quantity at its maximum corresponds to the K at the absolute ceiling.

It is
$$C_L^2 - \frac{C_D^{4/3}}{K^2} = 0 \tag{13}$$

$$K = \left(\frac{C_D^2}{C_L^3}\right)^{1/3} = \frac{1}{\left[\frac{C_L^3}{C_D^2}\right]} \tag{14}$$

So that the airplane can just maintain level flight if it is flown at $\left[\frac{C_L^3}{C_D^2}\right]_{max.}$ when K is the K of absolute ceiling.

The maximum value of the characteristic quantity can be found by differentiating it with respect to C_D and setting the result equal to zero.

$$\frac{d}{dC_D}\left[C_L^2 - \frac{C_D^{4/3}}{K^2}\right] = 0 \tag{15}$$

$$K = \left[\frac{\frac{2}{3}C_D^{4/3}}{C_L\, C_L'}\right]^{1/2} \tag{16}$$

Thus as K increases C_L' approaches zero and the asymptote of the curve of maximum values of the characteristic quantity is the line corresponding to $C_L' = 0$, the stall point.

As a practical matter, since the stall occurs in a wide region, for values of K above .5 the minimum value of R may be considered at the stall point. When K is very small, at high altitudes, the minimum radius of turn is obtained by flying at an angle of attack below the stall point.

No matter how large K becomes the maximum value of the characteristic quantity cannot exceed C_L^2. Thus, as K becomes large the influence of power on the radius of turn becomes less and less. For large values of K equation (11) may be written ---

$$R = \frac{W/S}{\frac{d}{2}\, g\, C_L} \quad \text{ft.} \qquad (11a)$$

So that for airplanes with large amounts of power, the general case up to the critical altitude, radius of turn is dependent only on wing loading, air density and C_L.

Equation (11a) is the limiting case where the angle of bank approaches 90°.

The quickest turn, as represented by equation (12) is dependent upon the wing loading, the air density and its characteristic quantity $K \frac{C_L^{\,2}}{C_D^{\,2/3}} - \frac{C_D^{\,2/3}}{K}$.

This characteristic quantity is a function of K and the polar.

The plot of the characteristic quantity for the quickest turn versus C_D is shown in Fig. 3.

Since the maximum $\bar{w}$ corresponds to the maximum $\left[K \frac{C_L^{\,2}}{C_D^{\,2/3}} - \frac{C_D^{\,2/3}}{K}\right]$ it is advantageous to fly at the values of C_D as shown on the heavy line on the graph.

The value of K which corresponds to zero rate of turn, or the absolute ceiling is

$$K \frac{C_L^{\,2}}{C_D^{\,2/3}} - \frac{C_D^{\,2/3}}{K} = 0$$

which yields

$$K = \left[\frac{C_D^{\,2}}{C_L^{\,3}}\right]^{1/3} = \frac{1}{\left[\frac{C_L^{\,3}}{C_D^{\,2}}\right]^{1/3}} \qquad (17)$$

as was obtained from minimum radius considerations.

8095

The maximum rate of turn occurs when

$$\left[K \frac{C_L^{2}}{C_D^{2/3}} - \frac{C_D^{2/3}}{K} \right] \quad \text{is a maximum}$$

or when

$$K \left[\frac{2 C_L \, C_L' \, C_D^{2/3} - {}^{2}/_{3} C_L^{2} \, C_D^{-1/3}}{C_D^{4/3}} \right] - \frac{{}^{2}/_{3} \, C_D^{-1/3}}{K} = 0$$

$$K = \left[\frac{C_D^{4/3}}{(3 \, C_L \, C_D \, C_L' - C_L^{2})} \right]^{1/2} \tag{18}$$

Thus as K increases $(3\ C_L\ C_D\ C_L' - C_L^2)$ approaches zero and the asymptote of the curve is the line corresponding to:

$$3 \, C_L \, C_D \, C_L' - C_L^{2} = 0 \tag{19}$$

$$\frac{C_D^{2}}{C_L} \left[\frac{C_L^{3}}{C_D} \right]' = 0 \tag{20}$$

which is the point on the polar where $\left[\frac{C_L^3}{C_D} \right]$ reaches its maximum value.

The quickest turn is flown at angles of attack which approach the angle of attack corresponding to

$$\left[\frac{C_L^{\,3}}{C_D}\right] \text{max.}$$

As a practical matter, for values of K. over .5 the maximum value of $\bar{w}$ may be considered to be at the

$$\left[\frac{C_L^{\,3}}{C_D}\right]_{max} \text{point.}$$

Very low values of K which occur at high altitudes require a reduction in the angle of attack.

Unlike radius of turn which becomes independent of K at large values of K, rate of turn is always dependent on K. For large K equation (12) becomes

$$\bar{w} = \left[\frac{\frac{d}{2}\, g^2}{W/S}\right]^{1/2} \left[\frac{C_L^{\,3}}{C_D}\right]^{1/3} \left[K\right]^{[illegible]} \quad [illegible] \qquad ([illegible])$$

[illegible] wil [illegible]

[illegible] alo-

[illegible]

For a fixed wing loading and altitude K may be considered an inverse function of the actual power loading where

$$K = \left[\frac{d}{2} \frac{1}{W/_S} \right]^{1/3} \left[\frac{1}{W/_P} \right]^{2/3} \qquad (10)$$

Combining equation (12a) and equation (10)

$$\bar{w} = \frac{(\frac{d}{2})^2 \, g^2}{(W/_S)^2 \, (W/_P)} \left[\frac{C_L{}^3}{C_D} \right]^{1/3} \text{ Rad}/_{\text{sec}} \qquad (12b)$$

which holds true only for large values of P and d. For most airplanes equation (12b) is a good approximation up to the critical altitude for full throttle operation.

Analysis of equation (12b) shows that wing loading affects rate of turn as the inverse square while power loading affects rate of turn only inversely.

The quickest turn and the sharpest turn are flown at different angles of attack. The quickest turn corresponds to the point on the polar where $\frac{C_L{}^3}{C_D}$ is a maximum and the sharpest turn to the point on the polar where C_L reaches its maximum. Since the value of C_L increases very slowly with large changes of C_D at angles of attack close to that for C_L max, the angle of attack corresponding to $\frac{C_L^3}{C_D}$ max is lower than that corresponding to C_L max.

Velocity is inversely proportional to the cube root of C_D as determined in equation (7). The radius of turn is inversely proportioned to the square root of the quantity $\left[C_L^2 - \frac{C_D^{4/3}}{K^2} \right]$ as determined in equation (11) and shown in Fig. (2).

This may explain one of the techniques of the dog fight. A successful pilot always flies at the angle of attack corresponding to the quickest turn. He can shoot down an opponent who attempts to achieve the minimum radius of turn, because both planes will have the same radius of turn within a few feet but the onw which flies at the angle of attack corresponding to the quickest turn will fly the faster. The pilot who flies faster will get into shooting position behind his opponent and make a kill.

CHAPTER III

PROPELLER DRIVEN AIRPLANES IN A SPIRAL

A power spiral may be a deliberate maneuver on the part of the pilot or may result inadvertently from a highly banked turn with insufficient power to maintain level turning flight.

In a power spiral equation (1) becomes

$$t + W \sin A_D = \frac{d}{2} S V^2 C_D \text{ lbs} \qquad (21)$$

where the angle A_D is measured to give a positive sense downward.

Equation (2) by including the effects of the deflection of the flight path from the horizontal becomes

$$L = \left[(W \cos A_D)^2 + F_c^2 \right]^{1/2} = \frac{d}{2} S V^2 C_L \text{ lbs} \qquad (22)$$

By [illegible] use of equation (3) [illegible] solving for R

$$R = \frac{W/S}{\left(\frac{d}{2} g\right) \left[C_L^2 - C_D^2 \left[\frac{[illegible] \cos A_D}{t + [illegible]} \right]^2 \right]^{1/2}} \text{ ft} \qquad ([illegible])$$

$$[illegible] = \frac{V}{R} \qquad ([illegible])$$

where V_z is the vertical velocity of descent.

Employing equation ([illegible]) and equation (24) to equation (21)

$$\frac{d}{2} S V^2 \, C_D = T + W \sin A_D$$

$$= \frac{P}{V} + W \, \frac{V_z}{V}$$

$$V = \left[\frac{P + W V_z}{\frac{d}{2} S \, C_D} \right]^{1/3} \text{ ft/sec} \qquad (25)$$

$$T + W \sin A_D = \frac{P + W V_z}{V}$$

$$T + W \sin A_D = \left[(P + W V_z)^2 \, \frac{d}{2} \, S \, C_D \right]^{1/3}$$

$$\frac{T + W \sin A_D}{W \cos A_D} = \left[\frac{(P + W V_z)^2 \, \frac{d}{2} \, S}{W^3 \cos^3 A_D} \right]^{1/3} C_D^{\ 1/3}$$

Let

$$K_S = \left[\frac{(P + W V_z)^2 \quad \frac{d}{2} \, S}{W^3 \cos^3 A_D} \right]^{1/3} \qquad (2[illegible])$$

a non-dimensional parameter.

Equation (23) may be re-written for small values of A_D

$$R = \frac{W/S}{\left(\frac{d}{2}\, g\right)\left[C_L^{\,2} - \dfrac{C_D^{\,4/3}}{K_s^{\,2}}\right]^{1/2}} \quad \text{ft} \tag{27}$$

Solving for $\bar{w}$

$$\bar{w} = \frac{V}{R} = \frac{\left[\dfrac{P + W\,V_z}{\frac{d}{2}\, S\, C_D}\right]^{1/3} \left(\frac{d}{2}\, g\right)\left[C_L^{\,2} - \dfrac{C_D^{\,4/3}}{K_s^{\,2}}\right]^{1/2}}{W/S}$$

$$\bar{w} = \left[\frac{\frac{d}{2}\, g^2}{W/S}\right]^{1/2} \left[K_s\, \frac{C_L^{\,2}}{C_D^{\,2/3}} - \frac{C_D^{\,2/3}}{K_s}\right]^{1/2} \text{rad/sec} \tag{28}$$

Equations (27) and (28) are the same as equations (11) and (12) for horizontal flight except that the parameter K_s has replaced K. From equations (10) and (26) the ratio of K_s to K is

$$\frac{K_s}{K} = \left[\frac{(P + W\,V_z)^2}{P^2}\right]^{1/3}$$

$$= \left[1 + \left(\frac{W}{P}\right) V_z\right]^{2/3} \tag{29}$$

which shows that the ratio is a function of the velocity of descent and the power loading.

This relationship is plotted in Figure 6.

In a spiral the parameter K_s bears the same relationship to the turn that K does in horizontal flight. K_s is always greater than K for a dive and the ratio of K_s to K is a function of V_s as shown in equation (29).

A diving spiral, therefore, always makes possible a sharper and a quicker turn than can be made in level flight with the same airplane at the same altitude.

By combining Fig. (3) and (4) with Fig. (6) the variations in radius of turn and angular velocity with dive or climb may be computed.

CHAPTER IV

EFFECT OF FLAPS ON THE TURNING PERFORMANCE OF PROPELLER DRIVEN AIRPLANES

The effect of the use of flaps on the turning performance of a propeller driven airplane can be determined by comparison of the polars of the airplane in the flaps up and flaps down condition.

The ceiling of the airplane is at the altitude when

$$K = \frac{1}{\left[\dfrac{C_L^{\,3}}{C_D^{\,2}}\right]^{1/3}} \qquad (14)$$

at altitudes below the ceiling K is greater than this value.

But if

$$\frac{1}{\left[\dfrac{C_{L_f}^{\,3}}{C_{D_f}^{\,2}}\right]^{1/3}} < \frac{1}{\left[\dfrac{C_L^{\,3}}{C_D^{\,2}}\right]^{1/3}}$$

that is, if

$$\frac{C_{L_F}^{3}}{C_{D_F}^{2}} > \frac{C_L^{3}}{C_D^{2}}$$

the airplane has a higher ceiling with the flaps extended than with the flaps retracted. This is rarely the case.

The radius of turn with flaps extended will be less than the radius of turn without flaps, if:

$$\left[C_{L_F}^{2} - \frac{C_{D_F}^{4/3}}{K^2}\right]_{max} > \left[C_L^{2} - \frac{C_D^{4/3}}{K^2}\right]_{max}$$

The right side is shown in Fig. (2). A curve similar to Fig. (2) must be computed for the flap condition.

When K is large it will be noted that this is principally a comparison of C_{L_F} max vs C_L max and flaps will always decrease the radius of turn. This is true at low altitudes.

At high altitudes when K becomes small flaps may or may not be an advantage.

The angular velocity with flaps extended will be greater than the angular velocity without flaps if,

$$\left[\frac{C_{L_f}^{\,2}}{C_{D_f}^{\,2/3}} K - \frac{C_{D_f}^{\,2/3}}{K}\right]_{max} \qquad \left[\frac{C_L^{\,2}}{C_D^{\,2/3}} K - \frac{C_D^{\,2/3}}{K}\right]_{max}$$

The right side is shown in Fig. (3). A set of curves similar to Fig. (3) must be computed for the flap condition.

At large values of K this becomes a comparison of

$$\left[\frac{C_{L_f}^{\,3}}{C_{D_f}}\right]_{max} \quad \text{vs} \quad \left[\frac{C_L^{\,3}}{C_D}\right]$$

CHAPTER V

NORMAL ACCELLERATION IN TURNS

$$N = \frac{F_r}{W} = \frac{L}{W} = \frac{\rho}{2}\,\frac{S}{W}\,C_L\,V^2 \quad \text{"g units"} \qquad (30)$$

Using equation (7) for V

$$N = \frac{\rho}{2}\,\frac{S}{W}\,C_L \left[\frac{P}{\frac{\rho}{2}\,S\,C_D} \right]^{2/3}$$

$$= K\,\frac{C_L}{C_D^{\,2/3}} = K \left[\frac{C_L^{\,3}}{C_D^{\,2}} \right]^{1/3} \qquad (31)$$

At the ceiling $N = 1$ and $K = K_{min}$.

$$K_{min} = \frac{1}{\left[\dfrac{C_L^{\,3}}{C_D^{\,2}} \right]_{max}} \qquad (32)$$

[illegible] m[illegible] for.

[illegible] n (1[illegible]) [illegible] the m[illegible]mum [illegible] of N for a given value of K occurs when $\left[\dfrac{C_L^{\ 3}}{C_D^{\ 2}}\right]$ is a maximum.

Thus the maximum accelleration does not coinside with either the quickest or the sharpest turns. The maximum accelleration occurs at an angle of attack lower than that for the quickest or the sharpest turns.

The speed for the maximum accelleration is greater than the speed for the sharpest for the quickest turns.

The quickest and the sharpest turns do not place the maximum strain upon the airplane.

A skillful pilot may, therefore, obtain maximum maneuverability from his airplane without subjecting it to the maximum stress.

The turn of maximum normal accelleration as well as the quickest and sharpest turns for a propeller driven airplane is shown in fig. 16.

CHAPTER VI

CALCULATIONS FOR NON-STEADY TURNS

If the airplane is flown from level flight into a turn a decelleration will result due to the increased drag in the turn. At the commencement of the turn this force can be calculated:

$$T + \frac{W}{g} a = \frac{d}{2} S \, C_{D_{turn}} V^2 \tag{32}$$

the thrust can be calculated from equation (1)

$$T = \frac{d}{2} S \, C_{D \, level} V^{2'}$$

solving for the decelleration

$$a = \frac{g}{W} \left[\frac{d}{2} S V^2 C_{D_{turn}} - \frac{d}{2} S V^2 C_{D_{level}} \right] \tag{33}$$

from equation (2) the following can be written

$$L_{level} \left[\frac{C_D}{C_L} \right]_{level} = \frac{d}{2} S V^2 C_{D_{level}}$$

$$L_{turn}\left[\frac{C_D}{C_L}\right]_{turn} = \frac{d}{2}\, S\, V^2\, C_{D\,turn}$$

so that

$$a = \frac{g}{W} L_{level}\left[\frac{L_{turn}}{L_{level}}\left[\frac{C_D}{C_L}\right]_{turn} = \frac{C_D}{C_L}_{level}\right] \qquad (33a)$$

from equation (30)

$$a = g\left[N\left[\frac{C_D}{C_L}\right]_{turn} - \left[\frac{C_D}{C_L}\right]_{level}\right] \qquad (33b)$$

and from equation (31)

$$a = g\left[K\, C_{D\,turn}^{\ 1/3} - \left[\frac{C_D}{C_L}\right]_{level}\right] \qquad (33c)$$

This shows that the decelleration experienced by an airplane as it is flown into a turn is a function only of the power parameter K, the $C_{D\,turn}$ and the $\left[\frac{C_D}{C_L}\right]$ at which it was being flown in level flight.

For the calculation of equation (33c) the values of C_D in level flight and in the turn can be obtained from Fig. (2). The C_L for level flight will be the C_L corresponding to the C_D of level flight and can be obtained from the polar.

CHAPTER VII

JET AIRPLANES IN LEVEL FLIGHT

Equations (4) and (5) will now be solved for a jet airplane.

Here

$$T = T_j \qquad \text{lbs.} \tag{33}$$

Where T_j is a constant if the engine is a rocket and T_j is a function of the air density if the engine is a turbo jet.

$$\frac{T_j}{W} = a \tag{34}$$

Where   is the inverse thrust loading and corresponds to the parameter K for propeller driven airplanes.

Equations (4) and (5) become

$$R = \frac{W/S}{\left(\frac{d}{2}\, g\right)\left[C_L^{\,2} - \frac{C_D^{\,2}}{B^2}\right]^{1/2}} \qquad \text{ft.} \tag{35}$$

$$\bar{w} = \left[\frac{\frac{d}{2}\, g^2}{W/S} \right]^{1/2} \left[B \frac{C_L^{\;2}}{C_D} - \frac{C_D}{B} \right]^{1/2} \text{rad/sec} \qquad (36)$$

Proceeding as for the propeller driven airplane:

Equation (35) is analyzed by plotting the characteristic quantity $\left[C_L^{\;2} - \frac{C_D^{\;2}}{B^2} \right]$ versus C_D for various values of B and is plotted in Fig. 8.

Since the minimum B corresponds to the maximum $C_L^{\;2} - \frac{C_D^{\;2}}{B^2}$ it is advantageous to fly at the values of C_D as shown by the heavy line on the graph.

The value of B which is just sufficient to give a zero value to the characteristic quantity at its maximum corresponds to the minimum B for flight

$$\left[C_L^{\;2} - \frac{C_D^{\;2}}{B^2} \right] = 0 \qquad (37)$$

$$B = \frac{C_D}{C_L} = \frac{1}{\left[\frac{C_L}{C_D} \right]} \qquad (38)$$

So that the airplane can just maintain level flight if it is flown at $\left[\frac{C_L}{C_D}\right]_{max}$ when B is the B of minimum flight.

The maximum value of the characteristic quantity can be found by differentiating it with respect to C_D and setting the result equal to zero.

$$\frac{d}{dC_D}\left[C_L^{\,2}-\frac{C_D^{\,2}}{B^2}\right]=0 \qquad (39)$$

$$2\,C_L\,C_L'-\frac{2\,C_D}{B^2}=0$$

$$B=\left[\frac{C_D}{C_L\,C_L'}\right]^{1/2}$$

Thus as B increases C_L' approaches zero and the asymptote of the curve of maximum values of the characteristic quantity is the line corresponding to $C_L' = 0$, the stall point.

As a practical matter, since the stall covers a region, for values of B above .3 the minimum value of R may be considered at the stall.

No matter how large B becomes the maximum value of the characteristic quantity cannot exceed C_L^2. Thus for large values of B equation (35) reduces to equation (11a)

$$R = \frac{W/S}{\frac{d}{2} g \, C_L} \qquad \text{ft.}$$

and the same conditions obtain as for a propeller driven airplane.

The quickest turn as represented by equation (36) may be analyzed by plotting its characteristic quantity $\left[B \frac{C_L^2}{C_D} - \frac{C_D}{B} \right]$ versus C_D for various values of B and is shown in fig. 9.

Since the maximum $\bar{w}$ corresponds to the maximum value of $\left[B \frac{C_L^2}{C_D} - \frac{C_D}{B} \right]$ it is advantageous to fly at the values of C_D as shown on the heavy line on the graph.

The value of B at which the airplane can just maintain level flight is

$$\left[B \frac{C_L^{2}}{C_D} - \frac{C_D}{B} \right] = 0 \tag{40}$$

$$B = \frac{C_D}{C_L} = \frac{1}{\frac{C_L}{C_D}} \tag{38}$$

as was obtained from minimum radius considerations.

The maximum rate of turn occurs when

$$\left[B \frac{C_L^{2}}{C_D} - \frac{C_D}{B} \right] \quad \text{is a maximum}$$

or when

$$B \left[\frac{2\, C_L\, C_L{}'\, C_D - C_L^{2}}{C_D^{2}} \right] - \frac{1}{B} = 0$$

$$B = \left[\frac{C_D^{2}}{2\, C_L\, C_L{}'\, C_D - C_L^{2}} \right]^{1/2} \tag{41}$$

Thus as b increases $(2\, C_L\, C_L{}'\, C_D - C_L{}^2)$ approaches zero and the asymptote of the curve is the line corresponding to

$$2\, C_L\, C_L{}'\, C_D - C_L{}^2 = 0 \qquad (42)$$

$$C_D{}^2 \left[\frac{C_L{}^2}{C_D} \right]' = 0 \qquad (43)$$

which is the point on the polar where $\left[\frac{C_L{}^2}{C_D} \right]$ reaches its maximum value.

Since a is fixed for a given airplane the best C_D and thus the best angle of attack for the quickest and the sharpest turns is fixed. These angles of attack are not functions of air density or speed.

The radius of turn of a jet airplane from equation (35) is seen to be a function of wing loading, altitude, thrust loading, and the form of the polar. For large b the radius of turn is a function of wing loading, altitude, and C_L.

The angular velocity of turn of a jet airplane from equation (3[illegible]) is seen to be a function of wing loading, altitude, thrust loading, and the form of the polar. For large B the angular velocity is a function only of wing loading, altitude, thrust loading and $\left[\frac{C_L^{\,2}}{C_D} \right]$.

For large B equation (3[illegible]) reduces to

$$\bar{w} = \left[\frac{\frac{\rho}{2}\, g^2}{W/S} \right]^{1/2} \left[\frac{C_L^{\,2}}{C_D} \right]^{1/2} \left[\text{[illegible]} \right]^{1/2} \text{rad/sec} \qquad (3\text{[illegible]})$$

CHAPTER VIII

EFFECT OF FLAPS ON THE TURNING PERFORMANCE OF JET AIRPLANES

The effect of the use of flaps on the turning performance of the jet airplane can be determined by comparison of the polars of the airplane in the flaps up and flaps down condition.

The ceiling of a jet airplane is at the altitude when:

$$B = \frac{1}{\left[\dfrac{C_L}{C_D}\right]} \tag{38}$$

at altitudes below the ceiling B is greater than this value.

But if

$$\frac{1}{\dfrac{C_{L_f}}{C_{D_f}}} < \frac{1}{\dfrac{C_L}{C_D}}$$

that is, if

$$\frac{C_{L_f}}{C_{D_f}} > \frac{C_L}{C_D}$$

The airplane has a higher ceiling with flaps extended than with flaps retracted. This is rarely the case.

The radius of turn with flaps extended will be less than the radius of turn without flaps, if,

$$\left[C_{L_f}^{\ 2} - \frac{C_{D_f}^{\ 2}}{B^2} \right]_{max} > \left[C_L^{\ 2} - \frac{C_D^{\ 2}}{B^2} \right]_{max}$$

The right side is plotted in Fig. (8). A set of curves similar to Fig. (8) must be computed for the flapped condition.

When B is large the comparison is principally one of $C_{L_f \ max}$ vs $C_{L \ max}$ and flaps will always decrease the radius of turn. This is true at low altitudes.

At high altitudes where B becomes small flaps may or may not be an advantage.

The angular velocity with flaps extended will be greater than the angular velocity without flaps, if

$$\left[B \frac{C_{L_f}^{\,2}}{C_D} - \frac{C_{D_f}}{B} \right]_{max} \qquad \left[B \frac{C_L^{\,2}}{C_D} - \frac{C_D}{B} \right]_{max}$$

The right side is plotted in Fig. (9). A set of curves similar to Fig. (9) must be computed for the flapped condition.

At large values of B this reduces to a comparison between

$$\left[\frac{C_{L_f}^{\,2}}{C_{D_f}} \right]_{max} \quad \text{vs} \quad \left[\frac{C_L^{\,2}}{C_D} \right]_{max}$$

CHAPTER IX

JET AIRPLANES IN A SPIRAL

The considerations of the general case of the spiral which lead to equation (43) are valid for the jet driven airplane.

$$R = \frac{W/S}{\left(\frac{d}{2}\, g\right)\left[C_L^{\,2} - C_D^{\,2}\left[\frac{W \cos A_D}{T + W \sin A_D}\right]^2\right]} \quad 1/2 \text{ ft} \qquad (43)$$

Using equations (21) and (34) and solving for V

$$T + W \sin A_D = \frac{d}{2}\, S\, V^2\, C_D \quad \text{lbs.} \qquad (21)$$

$$\sin A_D = \frac{V_z}{V} \qquad (24)$$

$$T = T_j = WB \quad \text{lbs.} \qquad (34)$$

$$T + W \sin A_D = \frac{d}{2}\, S\, V^2\, C_D$$

$$WB + W\, \frac{V_z}{V} = \frac{d}{2}\, S\, C_D\, V^2$$

$$V^3 - \frac{W\,B}{\frac{d}{2}\,S\,C_D}\,V - \frac{W\,V_z}{\frac{d}{2}\,S\,C_D} = 0 \qquad (44)$$

solving the cubic and restricting V_z to small values.

$$V = \left[\frac{W\,B}{\frac{d}{2}\,S\,C_D}\right]^{1/2} \quad \text{ft/sec} \qquad (45)$$

$$\frac{T + W\sin A_D}{W\cos A_D} = \frac{WB + W\,\frac{V_z}{V}}{W} = B + \frac{V_z}{V}$$

$$= B + \frac{V_z}{\left[\frac{W\,B}{\frac{d}{2}\,S\,C_D}\right]^{1/2}} = B + V_z\,\frac{\left[\frac{d}{2}\,\frac{S}{W}\,C_D\right]^{1/2}}{B^{1/2}} = B_s \qquad (46)$$

Calling

$$\frac{1}{B^2}\left[1 - \frac{V_z}{\left[\frac{W}{\frac{d}{2}\,S\,C_D}\right]^{1/2}}\right] = \text{[illegible]} \qquad \text{([illegible])}$$

Equation (23) becomes for the jet airplane

$$R = \frac{W/S}{\left(\frac{d}{2}\,g\right)\left[\dfrac{\;}{C_L^{2} - \dfrac{C_D^{2}}{B_s^{2}}}\right]^{1/2}} \quad \text{ft} \tag{51}$$

where B_s bears the same relationship to R in a spiral turn as B does in a horizontal turn.

Similarly for $\bar{w}$

$$\bar{w} = \frac{V}{R} = \frac{\left[\dfrac{W\,B}{\frac{d}{2}\,S\,C_D}\right]^{1/2} \left(\frac{d}{2}\,g\right) \left[C_L^{2} - \dfrac{C_D^{2}}{B_s^{2}}\right]^{1/2}}{W/S}$$

$$= \left[\frac{\frac{d}{2}\,g^{2}}{W/S}\right]^{1/2} \left[B\,\frac{C_L^{2}}{C_D} - C_D\,\frac{B}{B_s^{2}}\right]^{1/2} \text{rad/sec} \tag{52}$$

CHAPTER X

CONCLUSIONS

1. That the turning performance of airplanes engaged in military maneuvers at full power can be best described in terms of the concepts of sharpest turn and quickest turn; and that airplane velocity does not enter into the analysis since in full power flight it is not subject to direct control.

2. That insofar as design of a fighter airplane is concerned wing loading and the lift of the wings are the factors of primary importance in influencing turning performance.

3. That power loading or thrust loading has a very minor influence on radius of turn; and that at altitudes below 10,000 feet for present day fighters the radius of turn would be decreased a negligible amount by the addition of unlimited engine power.

4. That angular velocity of turn does depend on power loading or thrust loading as well as wing loading and the lift to drag characteristics of the wings.

5. That the turning performance of airplanes deteriorates with increasing altitude due to the decreased density of the air reducing the lift of the wings and power of the engine.

6. That the pilot influences the turning performance of his airplane by his selection of angle of attack in the turn; and that the optimum angle of attack for given conditions of power or thrust loading and altitude is shown in Figures 2, 3, 8, and 9.

7. That the angle of attack for optimum turning performance differs between the sharpest turn and the quickest turn; it also differs between propeller driven and jet airplanes.

8. That a diving spiral makes possible a sharper and a quicker turn than can be made in level flight; and that for a climbing spiral the reverse is true.

9. That the effect of flaps on turning performance can be predicted by comparison of the airplane polars for flapped and unflapped flight.

10. That the turn of maximum normal accelleration and force on the airplane does not coincide with either the sharpest turn or the quickest turn but occurs at a lower angle of attack and a greater speed.

11. That the decelleration experienced by an airplane as it is flown from straight flight into a turn is a function of the power parameter, the drag coefficient in the turn, and the lift to drag ratio at which it was being flown in level flight.

BIBLIOGRAPHY

"Airplanes in Horizontal Curvilinear Flight" by Heinrich Kann. N.A.C.A Technical Note No. 174, January 1924.

"Curvilinear Flight of Airplanes" by L. Salkowski. N.A.C.A. Technical Note No. 176, January 1924.

"Aircraft Turning Performance" by S. B. Gates, Aeronautical Research Committee Reports and Memorandum No. 1502, August 1932.

"Notes on the Dog Flight" by S. B. Gates. Royal Aircraft Establishment Report No. B. A. 1613, June 1940.

"Notes on the Turning Performance of the Spitfire as Affected by Altitude and Flaps" by M. B. Morgan and D. E. Morris. Royal Aircraft Establishment Report No. B. A. 1[illegible], April 1941.

"Study of Turning Performance of a Fighter-Type Airplane Particularly as Affected by Flaps and Increased Supercharging" by J. W. Wetmore. N.A.C.A Advance Confidential Report, June 1942.

"Calculated and Measured Turning Performance of a Navy F2A-3 Airplane as Affected by the Use of Flaps" by L. A. Clousing, B. L. Gadeberg, and W. M. Kauffman. N.A.C.A Advance Confidential Report No. 3130, September 1943.

"Performance Theory of Airplanes with Jet Propulsion" by Lippisch. Air Material Command Translation Report No. F-TS-685-RE, October 1946.

Figure 1 is a plot of the Parameter K versus Altitude for a typical propeller driven airplane. The curve represents flight with the engine at military power output. The area under the curve represents flight with the power output of the engine less than military power. The break in the curve represents the effects of two stage supercharging.

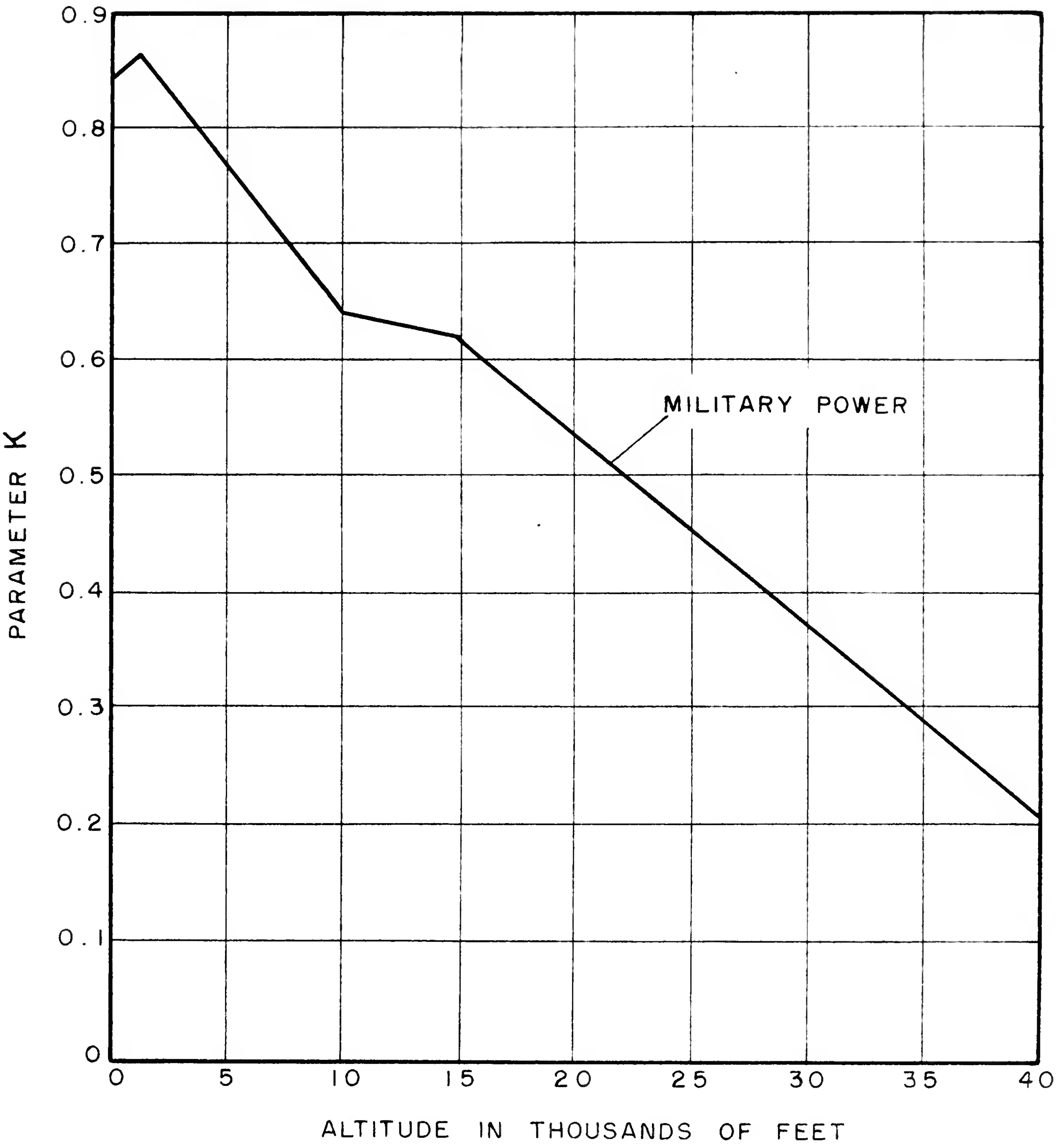

FIG. 1
PARAMETER K vs ALTITUDE
FOR
TYPICAL PROPELLER DRIVEN AIRPLANE

Figure 2 is a plot of the characteristic quantity for the sharpest turn versus the drag coefficient for a typical propeller driven airplane. Curves are drawn for various values of the Parameter K and from them the locus of their maximum values has been determined. The locus indicates the optimum flight condition for the airplane to fly at the minimum radius of turn at any value of the parameter K. The results are given in terms of C_D and must be converted to angle of attack by reference to the airplane polar. Unfortunately, the C_D versus angle of attack data on this airplane is not available.

The asymptote of the locus is the value of C_D corresponding to the maximum value of C_L.

The curves corresponding to K of .85 and unlimited K fall very close together. This shows that insofar as radius of the sharpest turn is concerned, power beyond that available at sea level has little effect.

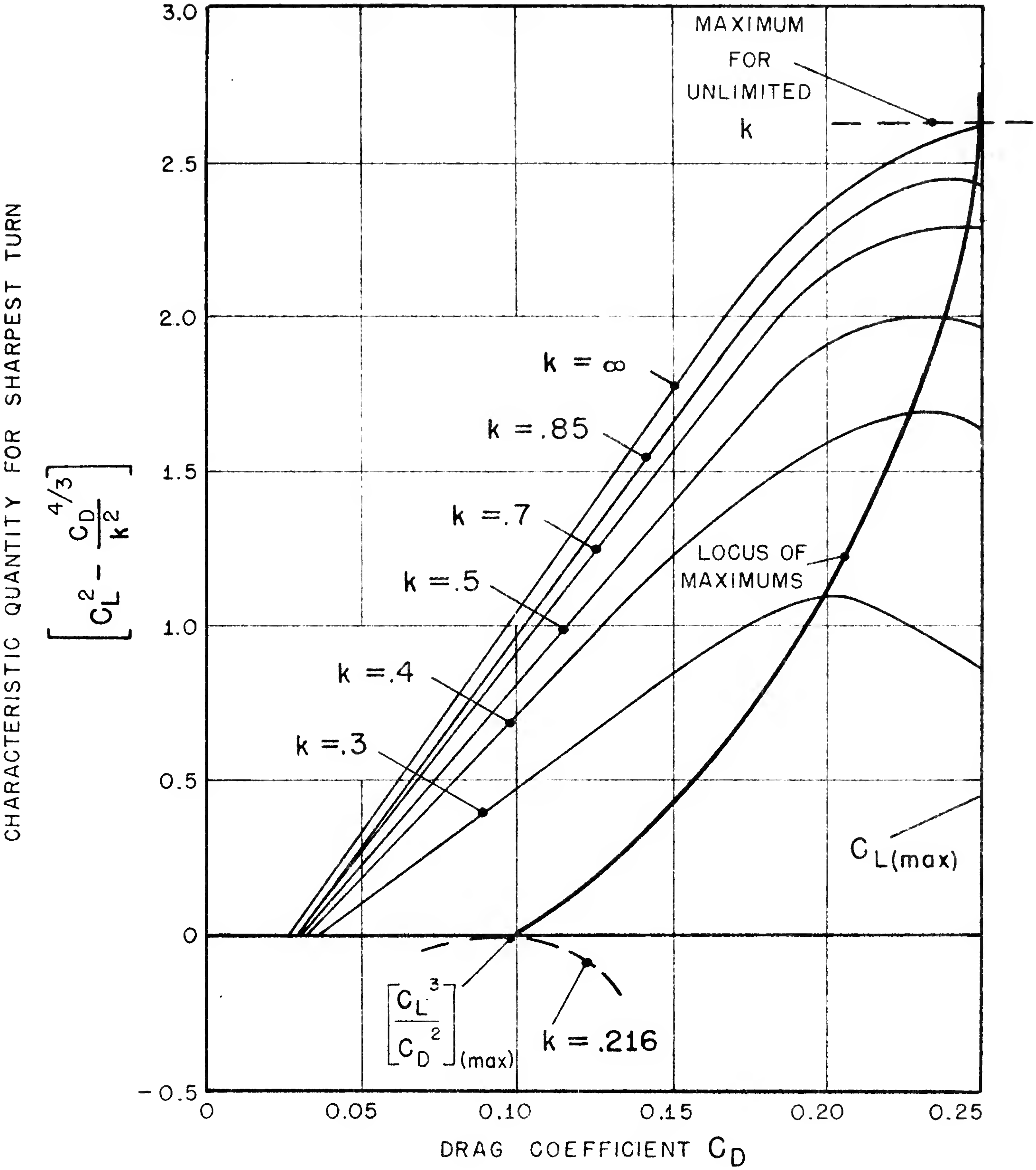

FIG. 2

CHARACTERISTIC QUANTITY FOR SHARPEST TURN

VS

DRAG COEFFICIENT

FOR

TYPICAL PROPELLER DRIVEN AIRPLANE

Figure 3 is a plot of the characteristic quantity for the quickest turn versus the drag coefficient for a typical propeller driven airplane. Curves are drawn for various values of the parameter K and from them the locus of their maximum values has been determined. The locus indicates the optimum flight conditions for the airplane to fly at maximum angular velocity of turn at any value of the parameter K. The results are given in terms of C_D and must be converted to angle of attack by reference to the airplane polar. Unfortunately, the C_D versus angle of attack data on this airplane is not available.

The asymptote of the locus is the value of C_D corresponding to the maximum value of the ratio C_L^3 to C_D.

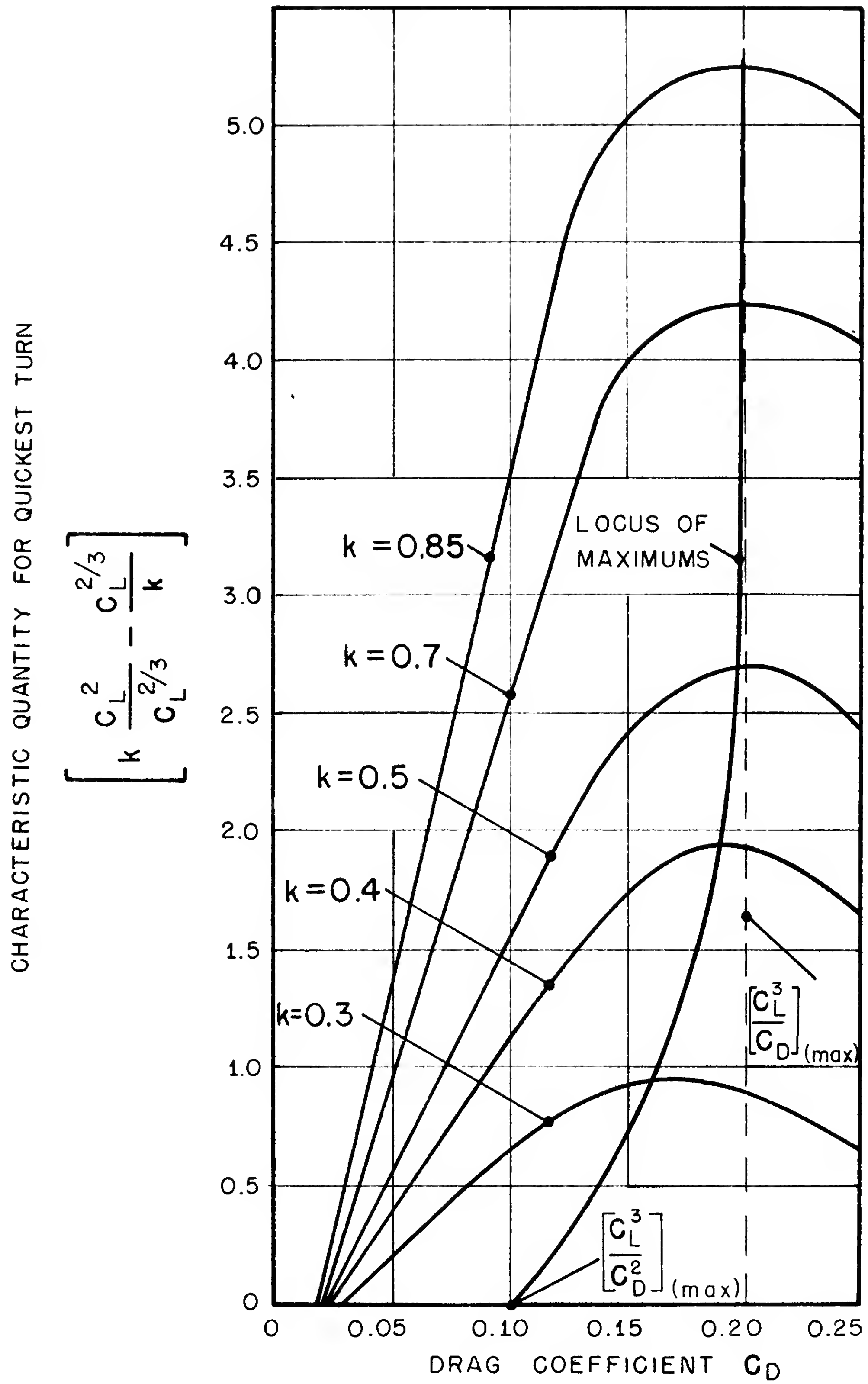

FIG. 3
CHARACTERISTIC QUANTITY FOR QUICKEST TURN VS
DRAG COEFFICIENT FOR TYPICAL PROPELLER DRIVEN AIRPLANE

Figure 4 is a plot of the minimum radius of turn versus altitude for a typical propeller driven airplane. The solid curve shows the relationship with military power output from the engine. The dotted curve shows what would be the relationship if the power output of the engine were increased without limit. Below 10,000 feet altitude the two curves lie very close together. Engine power is of minor importance in determining the minimum radius of turn at altitudes below 10,000 feet.

The asymptote of the military power curve is 39,000 feet altitude, the absolute ceiling of the actual airplane. The dotted curve extends indefinitely.

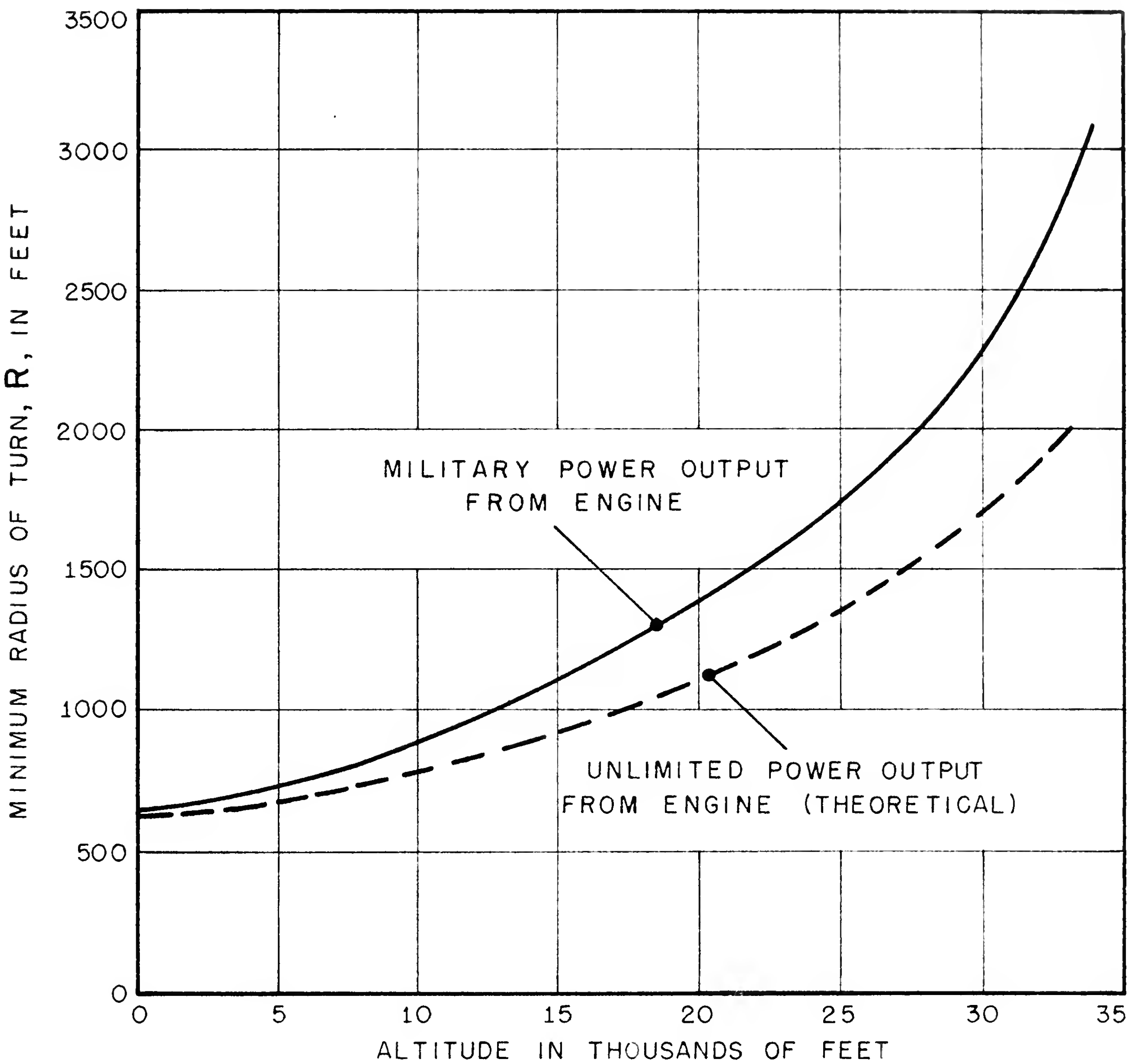

FIG. 4

MINIMUM RADIUS OF TURN
VS
ALTITUDE
FOR
TYPICAL PROPELLER DRIVEN AIRPLANE

Figure 5 is a plot of the maximum angular velocity of turn versus altitude for a typical propeller driven airplane. Except for the discontinuities introduced by engine supercharging, angular velocity has an almost linear relationship with altitude in the region where the locus of maximums is close to the asymptote (See Figure 3). When the locus of maximum corresponds to a lower value of C_D the angular velocity drops off rapidly with altitude and is zero at 39,000 feet, the absolute ceiling of this airplane.

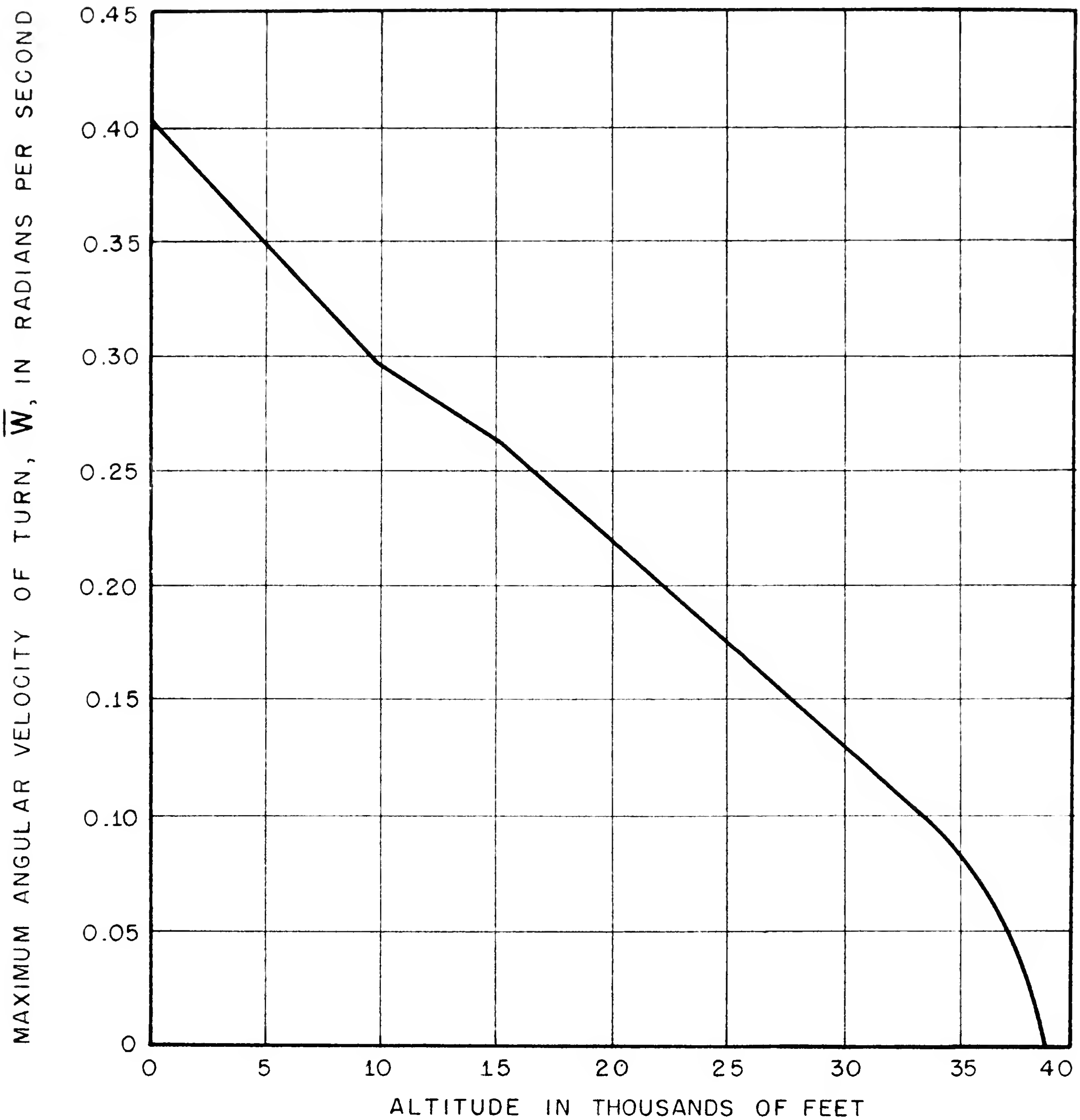

FIG. 5

MAXIMUM ANGULAR VELOCITY OF TURN

VS

ALTITUDE

FOR

TYPICAL PROPELLER DRIVEN AIRPLANE

Figure [illegible] is a plot of the ratio of the parameter K_s to the parameter K versus vertical velocity for a typical propeller driven airplane. Two curves are drawn to show the relationship at sea level and at 25,000 feet altitude with military power output from the engine. A dive is 1.[illegible] times as effective and a climb 1.[illegible] times as difficult at 25,000 feet altitude than at sea level in so far as the parameter K is concerned.

From Figure [illegible] and Figures 2 and 3 it can be calculated that a dive of 200 feet per second at 25,000 feet will decrease the radius of turn from 1725 feet to 1550 feet and increase the angular velocity of turn from .18 radians per second to .20 radians per second.

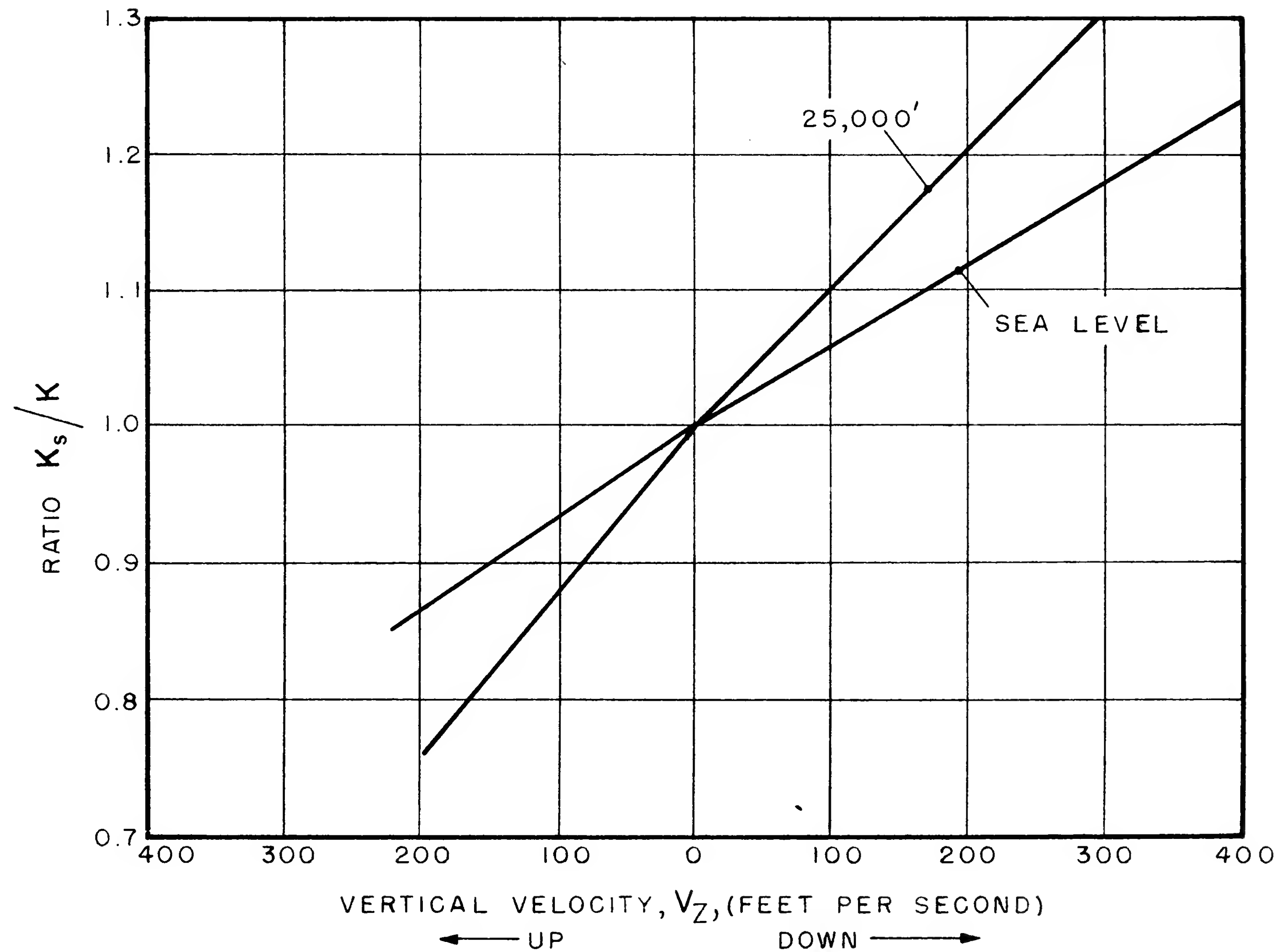

FIG. 6

RATIO $\frac{K_s}{K}$ vs VERTICAL VELOCITY, V_Z FOR A TYPICAL PROPELLER DRIVEN AIRPLANE AT FULL THROTTLE (MILITARY POWER)

Figure 7 is a plot of the parameter B, the inverse thrust loading, versus altitude for a typical turbo-jet airplane. The curve represents flight with the engine at military power output. The area under the curve represents flight with the power output of the engine less than military power.

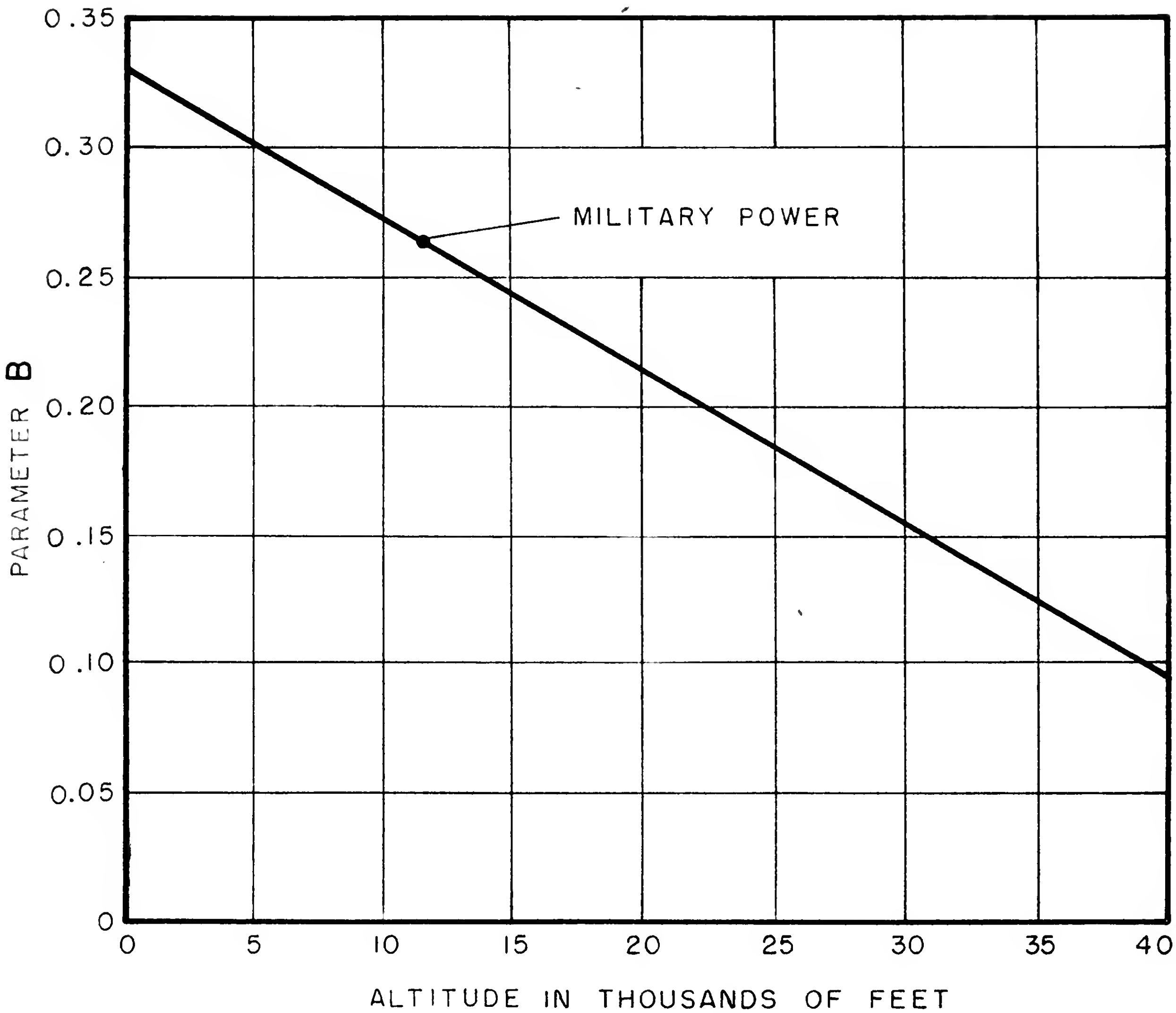

FIG. 7

PARAMETER B vs ALTITUDE

FOR

TYPICAL TURBO-JET AIRPLANE

Figure 8 is a plot of the characteristic quantity for the sharpest turn versus the drag coefficient for a typical turbo-jet airplane. Curves are drawn for various values of the parameter B and from them the locus of their maximum values can be determined. The locus indicates the optimum flight conditions for the airplane to fly the minimum radius of turn at any value of the parameter B. The results are given in terms of C_D be converted to angle of attack by reference to the airplane polar. Unfortunately, the C_D versus angle of attack data on this airplane is not available.

The asymptote of the locus is the value of C_D corresponding to the maximum value of C_L.

The locus curve has its origin at the maximum value of the ratio C_L to C_D. This corresponds to a lower angle of attack than for the propeller driven airplane (Figure [illegible]) where the locus has its origin at the maximum value of the ratio C_L^2 to C_D.

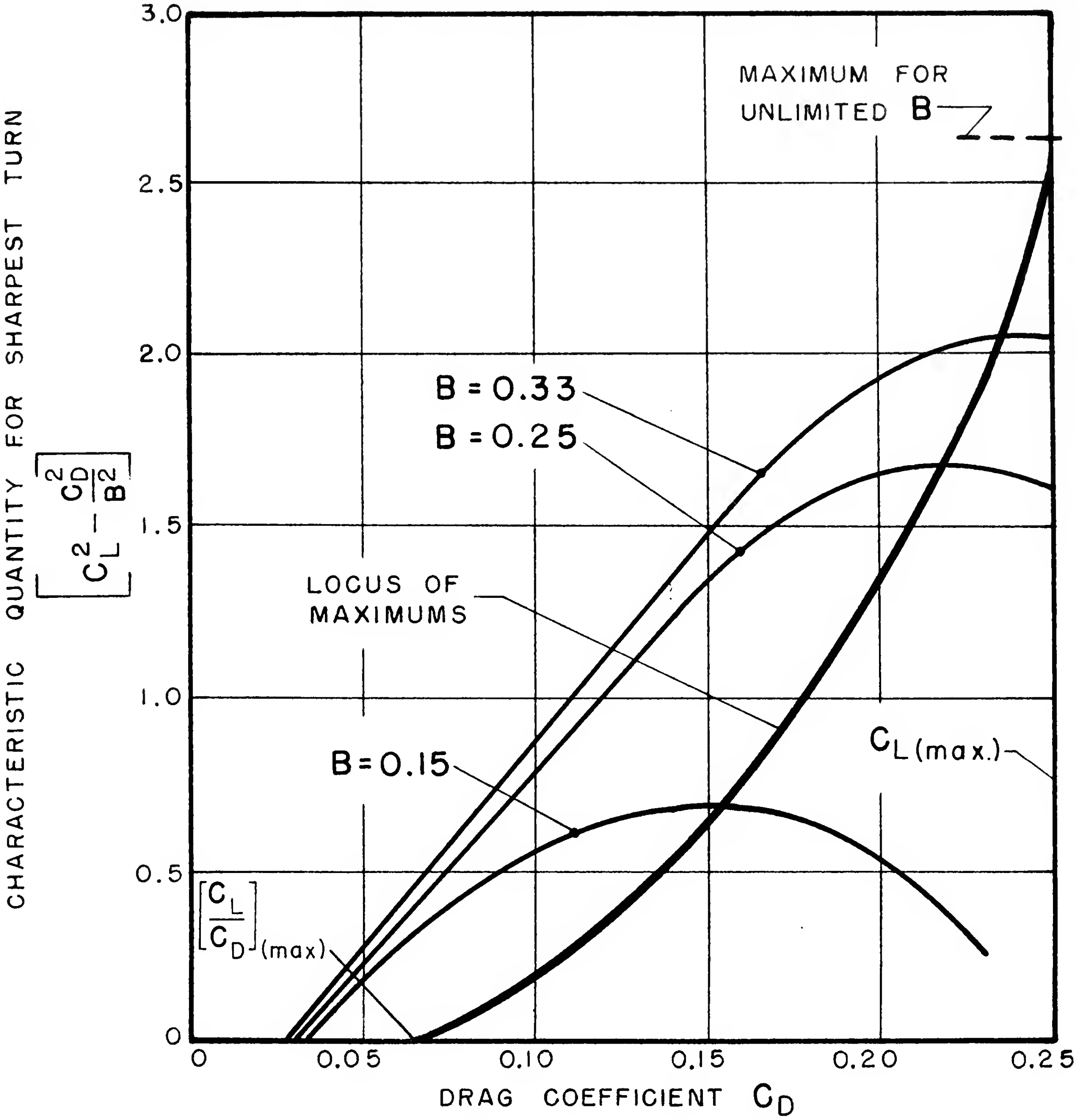

FIG. 8

CHARACTERISTIC QUANTITY FOR SHARPEST TURN

vs

DRAG COEFFICIENT

FOR

TYPICAL TURBO-JET AIRPLANE

Figure 9 is a plot of the characteristic quantity for the quickest turn versus drag coefficient for a typical turbo-jet airplane. Curves are drawn for various values of the parameter R and from them the locus of their maximum values has been determined. The locus indicates the optimum flight conditions for the airplane to fly at maximum angular velocity of turn at any value of the parameter R. The results are given in terms of C_D and must be converted to angle of attack by reference to the airplane polar. Unfortunately, the C_D versus angle of attack data on this airplane is not available.

The asymptote of the locus is the value of C_D corresponding to the maximum value of the ratio C_L^2 to C_D. The origin of the locus is at the value of C_L to C_D. The region of maximum for quickest turn of a turbo-jet airplane is smaller than that for a propeller driven airplane (Figure 3).

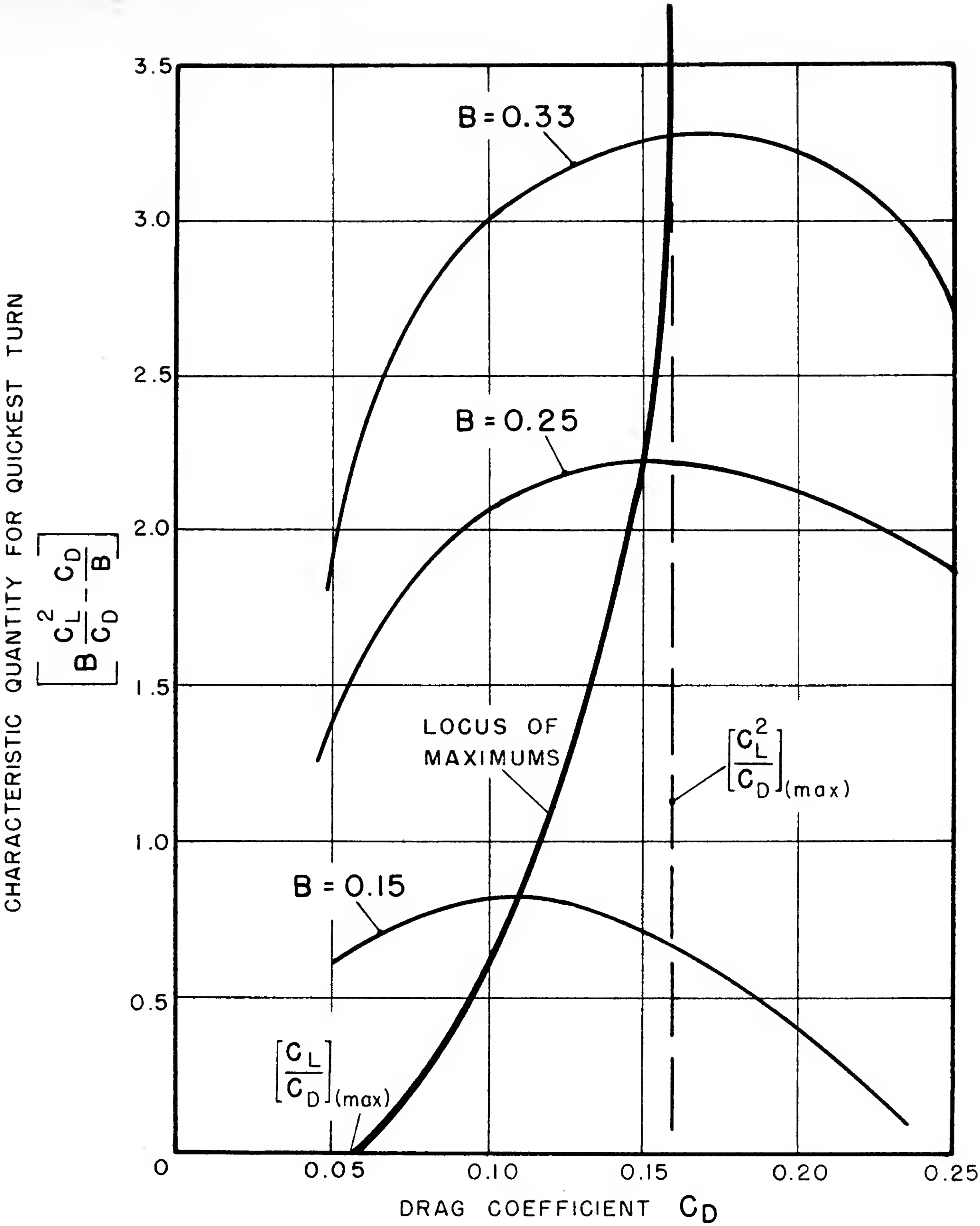

FIG. 9

CHARACTERISTIC QUANTITY FOR QUICKEST TURN

VS

DRAG COEFFICIENT

FOR

TYPICAL TURBO-JET AIRPLANE

Figure 10 shows a typical propeller driven airplane at sea level flying the sharpest turn, the quickest turn, and the turn with the maximum accelleration. Data on radius of turn, angular velocity of turn, true velocity, and normal accelleration for each turn is given. The advantages of flying at a high angle of attack are shown to be shorter radius of turn, greater angular velocity and reduced normal accelleration.

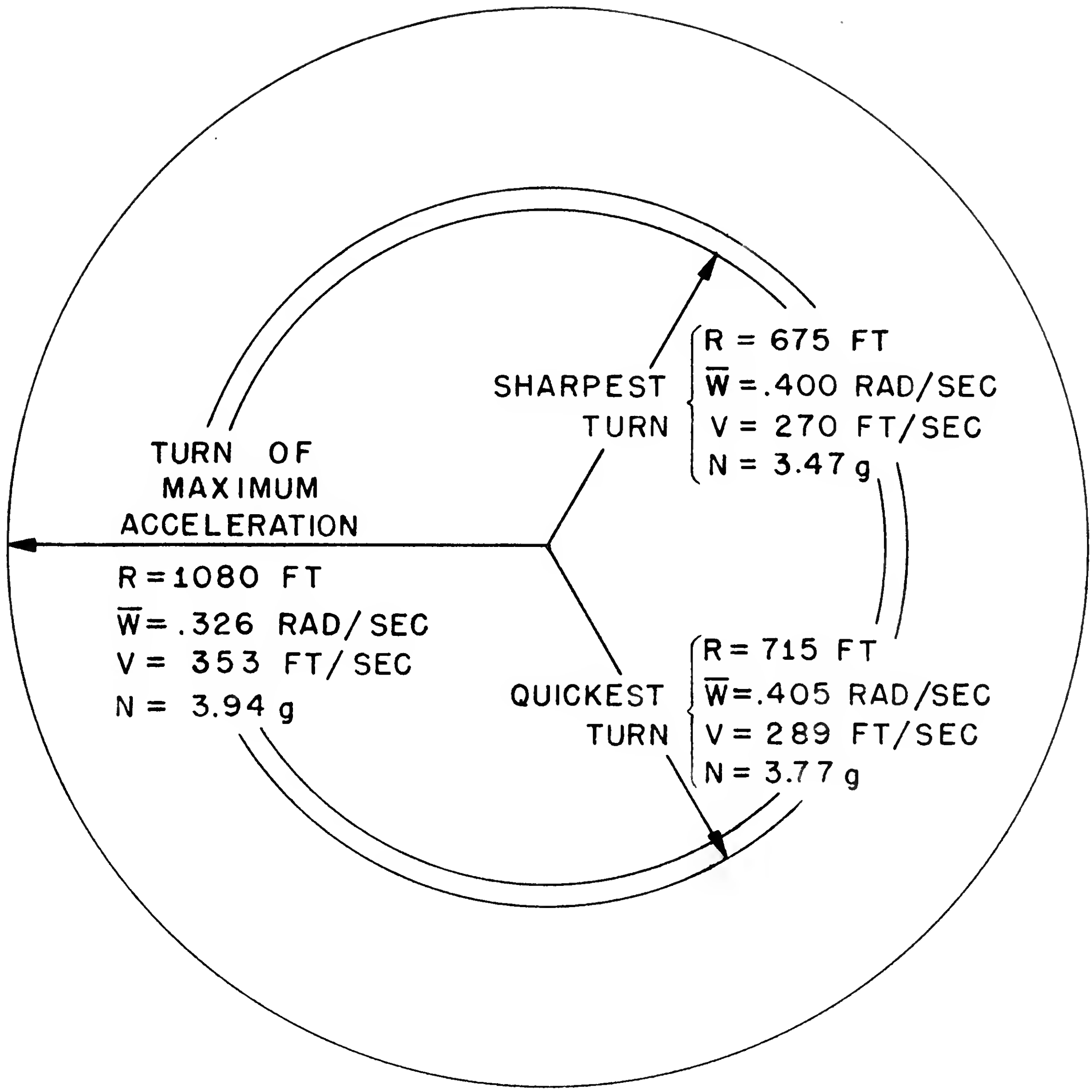

FIG. 10

THREE TURNS AT SEA LEVEL
FOR
TYPICAL PROPELLER DRIVEN AIRPLANE